

# Resit Probability Theory - 202001233 (2024-2025) for M4-TCS/BIT

## Version B

**Lecturers:** E. Castiel & A. K. Sinha,  
**Module Coordinator:** Faiza Bukhsh

**Last name, First name:** \_\_\_\_\_

**Student ID:** \_\_\_\_\_

**Instructions:**

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- This exam consists of 7 exercises.
- This document is your question as well as your answer paper.
- Write your answer in the designated boxes - try to keep your writing within the box.
- An extra box on the last page is given, in case you need extra space to finish a question. **You must write the question number** when you use the extra space on the last page.
- Additional scratch paper is available for your convenience (not graded!)
- The formula sheet and the probability tables are provided separately. You must not write on them and should return them after the exam is over.
- An ordinary calculator is allowed, not a programmable one (GR).
- Report numerical values to 3 decimal places where appropriate, unless mentioned otherwise.
- Please make your handwriting legible. Write only with **blue** or **black ink**. If you are using pencil, make sure that you use dark colour - light colour writing are very difficult to read. If we cannot read your writing, we will not award points.
- Make sure to write the number communicated to you when you sign the presence sheet on the top right of this page.

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**Do not use this page for computation or answering any question.**  
**Only write your name and student ID in the designated space above.**

1	2	3	4	5	6	7	Total
2	3	2	3	4	8	8	30

## Part 1.

*In part 1, only the final answer is required. You do not need to show your calculations.*

### Exercise 1: [2 Points] 1 for each answer

We throw an unbiased dice 100 times. For each  $k$ ,  $X_k = 1$  if the result of the  $k^{th}$  throw was a 1. We define  $\bar{X}_n = \sum_{k=1}^n X_k$ . Answer each question by clearly writing either **True** or **False** inside the boxes

1.  $Var(\bar{X}_{100}) \approx 13.889$
2.  $36 \frac{\bar{X}_{100} - \frac{500}{6}}{50}$  can be approximated by a standard normal random variable  $N(0, 1)$

### Exercise 2 [2 Points] All 4 correct: 2pt ; 3 correct: 1pt; 2 correct: 0.5pt; otherwise 0 pt.

For each of the following statements, write clearly either **True** or **False** inside the boxes.

Let A and B be two events.  $\bar{A}$  and  $\bar{B}$  represent the complements of A and B respectively. We know that  $P(A) = 0.7$ ,  $P(B) = 0.4$  and  $P(A | B) = 0.3$ .

1. A and B are independent

2.  $P(A \cup B) = P(A) + P(B)$

3.  $P(\overline{A \cup B}) = 0.15$

4.  $P(\bar{A} | B) = 0.7$

### Exercise 3: [3 Points] 1 for each answer

Let  $X$  and  $Y$  be two independent uniform discrete random variables. We have  $S_X = \{-3, -2, -1, 0, 1, 2\}$  and  $S_Y = \{-1, 0, 1, 2, 3, 4, 5\}$ . Answer the following, **round your answers to 3 decimal places, where appropriate**:

1.  $P(X \leq 0) =$

2.  $E[X + Y] =$

3. What is the support of the product between  $X$  and  $Y$  ( $S_{XY}$ )=

**Exercise 4: [3 Points], 1 for each answer**

Let  $X$  and  $Y$  be two independent normal random variables. We have  $E[X] = 1$ ,  $E[Y] = 2$ ,  $Var(X) = 16$ ,  $Var(Y) = 4$ . Round the answer from the table to the third decimal place where necessary.

1.  $P(\frac{Y-2}{2} \geq 0.5) =$

2.  $P(X \leq 3) =$

3.  $Var(3(X - Y)) =$

**Part 2.**

*In part 2, you do need to justify your answers mathematically. You have to show your calculations.*

**Exercise 5:[4 Points]**

**Fully justify and motivate your answers. You must show the necessary calculations. When necessary, round your answers to the third decimal.**

We throw two fair six sided die at the same time. Let  $X_1$  be the result of the first dice and  $X_2$  be the result of the second dice. Let  $X = \frac{7X_1}{2} - X_1X_2$ .

1. Calculate  $E[X]$ . Round your answer to the third decimal place [2 Points]

2. Calculate  $Cov(X, X_1)$ . Are  $X_1$  and  $X$  independent? *Round your answer to the third decimal place* **[2 Points]**

**Exercise 6:** **[8 Points]**

**Fully justify and motivate your answers. You must show the necessary calculations. Round your answers to the third decimal, when appropriate.**

The discrete random variable  $X$  has a probability density function (pdf)  $f_X$  given by

$$f_X(x) = \frac{e^{-x}}{1 - e^{-2}} \text{ for } 0 \leq x \leq 2$$

1. Find the numerical value of  $P(X < 1)$ . **[2 Points]**

2. Calculate  $E[X]$  and  $E[e^X]$  [**3 Points**]

3. Let  $Y = |X - 1|$ , where  $|\cdot|$  is the absolute value. Determine the range  $S_Y$  and the probability density function  $f_Y$ . [**3 Points**]

*Part 2. continues on the next page.*

**Exercise 7: [8 Points]**

Express your exact answer as a fraction without simplifying. You can use standard mathematical notations used in this course. Fully justify your answers and show necessary calculations. Define events to clearly indicate which probability you are computing.

A deck is composed of 40 cards numbered from 1 to 10. Each rank from 1 to 10 has 4 indistinguishable cards (there are four 1, four 2, etc...). We pick five cards uniformly at random.

1. What is the probability that exactly three cards among the five have the same rank? [2 Points]

2. Given that three of the cards are 1, 2 and 3, What is the probability that all 5 cards are consecutive numbers? [3 Points]

*Part 2. continues on the next page.*

3. Compute the expectation of the sum of the ranks of all five cards. [*3 Points*]

*Extra writing space on the next page*

*Extra writing space (can be used for any question as needed). Clearly write the question number which you are answering*